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DECISION-MAKING SUPPORT IN HUMAN RESOURCE MANAGEMENT BASED ON MULTI-OBJECTIVE OPTIMIZATION

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Abstract. The objective of this research is to develop a methodological approach to making managerial decisions in human resource management tasks. The paper substantiates the expedience of using multi-objective optimization, based on the Technique for Order of Preference by Similarity to Ideal Solution, to (or "intending to") improving the efficiency and transparency of human resource management decisions. A modified Technique is suggested to ensure the adaptability of multi-criteria decision-making in human resource management tasks. This modification consists in the integration of additional components into the decision algorithm. The method was tested during the solution of the real problem of selecting the best candidate among the applicants for the vacancies at the State Oil Company of the Azerbaijan Republic Human Resource Management Department. The results of the experiment showed the practical applicability and efficiency of the suggested approach for the objective and adequate evaluation of professional, qualification, and personal qualities of applicants, and for the support of managers during decision-making in the selection of personnel by the long-term tasks of the organization.

Keywords: decision making support, human resource management, TOPSIS, fuzzy environment, multi-objective optimization.

AMS Subject Classification: 68U35, 90B50.

1. Introduction

During the transition to a knowledge-based economy, the organization's (enterprise's, company's, firm's) efficient activity and competitiveness becomes significantly dependent on the human factor and the correct choice of the human resource management (HRM) policy [30, 38]. At the same time, globalization and rapid change of technologies precondition changes in the labor market, which, in its turn, causes considerable transformations in personnel relations, and requires the development of new conceptual approaches and scientifically substantiated methods in the policy that regulates these relations, depending on a specific HRM task. According to this concept, HRM is a special type of managerial activity. In this case, the main managed object is the human and his competencies, including knowledge, skills, and professional abilities, personal and behavioral qualities, motivational principles, intellectual and qualification potential, while HRM is aimed at supporting the organization's activity strategy under the growing role and importance of the human factor [5]. Therefore, in order to make decisions that are adequate to the new conditions with regard to personnel planning, selection, recruitment, adaptation to the changing market environment, retention, dismissal, promotion, development, training, and motivation of personnel, the decision-maker should evaluate and consider a wide range of information regarding the competencies of employees, be able to compare applicants, based on a multiple heterogeneous attributes (criteria), select the optimal solution (candidate) with the

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consideration of multiple influences, preferences, interests, and possible consequences. All these peculiarities of HRM tasks determine their multi-objectivity. At that, one should consider the volume, quantitative and qualitative nature, complexity and inconsistency of the flow of information that the decision-maker receives, which allow identifying HRM tasks as semi-structured tasks, for which the construction of objective models is either impossible or extremely difficult. Along with the abovementioned problems that arise during the generation and selection of managerial decisions, one should consider the decision-maker's preferences, and experts' competence (knowledge, intuition, experience, etc).

The fundamental bases of multi-criteria decision-making and certain applications of intelligent systems for decision-making support in HRM are provided [11, 24, 31, 47]. The main problems of multi-criteria decision-making tasks include the means of obtaining information, its nature and type, the methods of its presentation and processing, the determination of the number of considered variants (alternatives) and the number of their descriptive attributes, the hierarchal structure of the latter, the technologies of presenting expert knowledge, etc. At that, the human ability to make fewer mistakes while working with verbal data requires the selection of methods of handling linguistic variables. Therefore, in human resource management tasks, the handling of such data requires the application of models and methods, based on the fuzzy set and fuzzy logic theories [21, 44]. In order to overcome the abovementioned difficulties one needs to select, create, and develop methodological approaches to multiple-criteria analysis and decision-making in human resource management, based on intelligent technologies, methods, and computer systems of decision-making support [40].

The objective of this research is to develop a methodological approach to making managerial decisions in HRM tasks, which have such specific features as multi-objectivity and heterogeneity of data, the hierarchal, quantitative, and qualitative nature of criteria, their ambiguity, the need for considering the expert evaluation of their weight, and the influence of the experts' competence on the made decision.

2. Multi-criteria methods of decision-making in HRM tasks: literature survey

The literature survey shows among the HRM tasks' decision-making processes that require exceptional support, including employment management, assessment, and organization of personnel remuneration system, career planning, the formation of the reserve. Authors pay most attention to the selection and recruitment of staff resources which is caused by the greatest practical applicability of the latter. At present, while solving personnel selection tasks, developers mostly prefer multi-criteria methods of decision-making [8]. It includes methods of decision tree analysis [7], analytic hierarchy process (AHP) [12, 14, 41], the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [6], expert systems [2, 31], etc.

For example, [8] uses data analysis, based on a decision tree and association rules to (or "intending to") developing an efficient mechanism of forming rules on personnel selection for high technology companies. Without prejudice to the advantages of this approach, it is worth noting such drawbacks of the latter as the impossibility to generate rules in semi-structured areas that require expert knowledge, and the difficulty of building an optimal decision tree. [7] suggests a personnel selection system, based on the fuzzy analytic hierarchy process (FAHP), which allows evaluating alternatives by both qualitative and quantitative criteria. Also, uses fuzzy logic and AHP intending to reducing the subjectivity during the evaluation of personal qualities and essential professional skills of job candidates. According to the authors of works [7, 12, 41], the FAHP-based computer system of decision-making support eliminates restrictions to information on candidates and aids managers in making optimal decisions (selecting the best candidate) under fuzzy conditions. However, although these approaches allow making the best decision among the possible ones, they do not permit selecting an alternative that is preferable by all criteria, i.e. is the most similar to the ideal (optimal) solution. This possibility is provided by the TOPSIS method, which was first suggested by work [6].

Further, this method has found its application in various decision-making tasks, taking into account the specifics of which, the researchers have suggested various modifications. These changes are mainly reduced to:

(1) The introduction of group decision making [10, 19, 32, 33];
(2) The accounting for the hierarchical structure of criteria and their relative importance [29, 37, 46];
(3) The introduction of new metrics to calculate the distance to the ideal active and perfect negative solutions [2, 18].

However, it should be noted that the modification of TOPSIS method, which was mentioned above, affects only on certain aspects of the particular features of HRM tasks. In this paper, authors attempt to take into account all the components of the decision-making process in HRM functions.

The flexibility of the TOPSIS method, which provided the possibility of introducing additional stages and elements, led to the preference in choosing the latter as a method of multi-criteria optimization problems in HRM. In his way, the proposed integrated approach of HRM tasks solving simultaneously takes into account following:

(1) The possibility of group decision making;
(2) The hierarchical structure of the criteria;
(3) The relative importance of the criteria;
(4) The competence of experts participating in the evaluation process of the alternatives.

It should be noted that although the decision maker tries to select experts with approximately equal skill, however, in practice, it is challenging to meet this condition, and the preferences of experts in varying degrees, affect the decision. Moreover, although experts are a major figure in the HRM decision support system and their level of competence to some extent affects the final result. However, evaluation of scientific expertise involved in the decision-making process is not given due attention in the literature, and this problem remains poorly developed [1].

3. Materials and methods

3.1. **Specificity of human resource management tasks and their generalized conceptual model.** The specific features of HRM tasks for the intelligent support are determined [23, 25-28]. For example, the tasks, the solution whereof comes down to making efficient decisions, include the task of managing employment processes (selection, evaluation, and recruitment), the task of assessment (determining the conformity of personnel to the held position), the task of organizing a personnel remuneration system, the task of employees' career planning (promotion), the task of forming a reserve, etc. The analysis of mentioned HRM tasks allowed distinguishing the following peculiar features of the latter:

(1) multi-objectivity and heterogeneity of data that describe HRM tasks;
(2) the multilevel hierarchal structure of criteria, expressed in the fact that each individual upper-level criterion is based on the aggregation of partial criteria of the next lower level;
(3) quantitative and qualitative nature of criteria;

(4) the impossibility of unambiguous determination of criteria and the variableness of their value range;
(5) different extent of the influence of criteria and indicators on the considered variants (objects, alternatives), and the need to consider the difference in their weights. This determines the need for involving experts (information carriers) in decision-making, and the consideration of their opinions;
(6) the influence of the experts' competence on the quality of the made decision;
(7) the presence of a vast number of heterogeneous partial criteria in real situations, which complicate the formal comparison of alternatives.

The abovementioned peculiarities of HRM tasks allow identifying them as tasks of multiple-criteria analysis and decision-making in a fuzzy environment. Multiple-criteria analysis is generally required when solving such tasks as choice, evaluation, comparison, selection, ranking, and classification of objects (alternatives) in a fuzzy environment. These tasks are among the most common ones in decision-making support systems, and are encountered in various combinations [9, 36].

The analysis of currently developed approaches to and methods of solving HRM tasks demonstrated their diversity and allowed emphasizing that this situation is preconditioned by the following factors: 1) the formulation of the decision-making task in HRM; 2) the level of complexity of the set task, i.e. the extent of consideration of human resource management tasks' specificity (partial or full) in the formulation of such tasks; 3) the substantial and quantitative difference between the sets of criteria that characterize HRM tasks and partial criteria that influence the calculation of the integral indicator; 4) the difference in partial criteria's units of measurement and the methods of evaluating their weight (subjective, objective); 5) means of aggregating partial criteria; 6) the use of various methods of criteria convolution; 7) the need for the participation of experts in decision-making, or vice versa.

Thus, when selecting this or that method for the solution of abovementioned HRM tasks, one should be guided by the maximal consideration of abovementioned specific features of the latter. At the same time, the selected methodological approach should ensure:

(1) the absence of restrictions to the number of alternatives, criteria, and partial criteria;
(2) the evaluation of the competence of experts, who participate in decision-making;
(3) the possibility to extend the applied methodological approach to all HRM tasks that require intelligent support.

Based on the comprehensive approach to the consideration of the specificity of human resource management, the generalized conceptual model of decision-making in HRM tasks can be presented by the following set of information:

$$M_{HRM} = (X, K, Y, E, V, P, L, W),$$

where:

(1) $X = \{x_i,\ i = \overline{1,n}\}$ is the set of admissible alternatives;
(2) $K = \left\{K_j,\ j = \overline{1,m}\right\}$ is the set of choice criteria that characterize alternatives;
(3) $K_j = \left\{k_{jt},\ t = \overline{1,T}\right\}$ is the set of sub-criteria that characterize each criterion;
(4) Y is the range of definition of each partial criterion's value;
(5) E is the group of experts, participating in decision-making;
(6) V is the set of relation between experts in accordance with the decision-maker's preferences;
(7) P is the relations between the X, K and E sets;

(8) L is the linguistic expressions that reflect the level of partial criteria's satisfaction by alternatives (membership level);
(9) W is the relations between criteria and partial criteria.

According to the conceptual model, the essence of decision making in HRM tasks is:

1) finding a systematized list of alternatives ($X \longrightarrow X^*$), ranked from the best (optimal) one to the worst one (or vice versa);

2) choosing the best (optimal) variant from among the alternatives.

For this purpose, it is necessary to reduce a multi-objective optimization task to a single-objective one.

Typically, it is difficult to solve the multi-criteria optimization problem without some simplifications because of the complexity of criterion space. To simplify the procedure of multi-criterion alternatives comparison, it is necessary to aggregate a large number of unequal criteria with corresponding domains of the values definition of the individual criteria according to the preference of the experts, taking into account the competence degree of the latter. Formally it can be expressed as follows:

(1) fuzzy relations between sets of alternatives, criteria of evaluation of alternatives, groups of experts who evaluate how much the alternatives satisfy the criteria, considering the linguistic nature of the latter: $X \times K \times E \times Y \to p(x_i)_{i\times j\times t\times l}$.
(2) fuzzy relations between criteria: $K_j \times K_j \to w_j$ and partial criteria: $k_{jt} \times k_{jt} \to w_{jt}$;
(3) fuzzy relations of experts' competence, according to the decision-maker's preferences $E \times E \to v_l$.

With the specific peculiarities of decision-making tasks in HRM and the suggested conceptual model in mind, it is necessary to formulate in general the task of multi-objective ranking/choice of alternatives. A multi-objective optimization task is generally understood as finding the maximum and minimum vector-valued criterion in a feasible set of alternatives.

General formulation.

Given the following components of HRM tasks in organization:

1. $X = \{x_i,\ i = \overline{1,n}\}$ is the set of alternatives;
2. $K = \left\{K_j,\ j = \overline{1,\ m}\right\}$ is the set of criteria;
3. $K_j = \left\{k_{jt},\ t = \overline{1,\ s_j}\right\}$ is the set of partial criteria;
4. $E = \left\{e_l,\ l = \overline{1,\ g}\right\}$ is the set of experts;
5. $w_j,\ j = \overline{1,m}$ is the coefficients of criteria's relative importance ($K = \left\{k_j,\ j = \overline{1,\ m}\right\}$);
6.$w_{jt},\ t = \overline{1,\ s_j}\ ,j = \overline{1,m}$ is the coefficients of partial criteria's relative importance ($k_j = \left\{k_{jt},\ t = \overline{1,\ s_j}\right\}$);
7. $v_l,\ l = \overline{1,g}$ is the experts' competence coefficients.

Assume $f(x)$ is an objective function that guarantees the choice of the best alternatives:

(1) $f(x) = \max\left(f(x_1),\ f(x_2), ..., f(x_n)\right) \quad f(x) \to [0,\ 1]$,

where $f(x_i)$ is the resultant vector of the evaluation of alternative $x_i \in X$ in accordance with integral criterion *K*, i.e. $f(x_i) \to K(x_i)$.

(2) $K(x_i) = \ (p(x_i), w, v)$ is the integral evaluation of alternative x_i according to the set of evaluation criteria, the weight of partial criteria in the integral criterion K and the coefficient of the relative importance of experts' competence, where

- $p(x_i)$ is the integral evaluation of alternative $x_i,\ i = \overline{1,n}$ in accordance with the values of the linguistic variable by the experts' preference;

- $w = (w_1, ..., w_Z)$ are the weights of partial criteria in the integral criterion *K*, $z = \overline{1,Z}$. Z is the total number of partial criteria;

- $v = (v_1, ..., v_g)$ is the coefficient of the relative importance of experts' competence, according to the decision-maker's preferences.

(3) $f(x_i) > 0$, provided that $p(x_i) > 0$.

(4) $g(K(x), w, v) \in G,\ x \in X,$

(5) $w_j > 0,\ j = \overline{1, m},\ \sum_{j=1}^{m} w_j = 1,$

(6) $w_{jt} > 0,\ t = \overline{1,\ s_j}\ ,\ \sum_{t=1}^{s_j} w_{jt} = 1,$

(7) $w_z > 0,\ z = \overline{1,\ Z},$

(8) $v_l > 0,\ l = \overline{1,\ g},\ \sum_{l=1}^{g} v_l = 1.$

It is necessary to find the alternatives that best correspond with the objective functions and restrictions.

According to the task's formulation, the set of feasible solutions is formed by eliminating from the initial set of alternatives the ones that do not satisfy the set objective and restrictions.

As shown above, multi-objective HRM tasks are semi-structured problems that contain many criteria (both qualitative and quantitative) of decision quality evaluation. The decision-maker is guided by his or her subjective preferences with regard to the efficiency of possible alternatives and the importance of different criteria. Constructing a decision-maker preference model produces a large volume of information. It is difficult and, sometimes, impossible to estimate efficiency indicators and choose a single best decision by analytical methods. Therefore, the existing concepts of evaluating preferences are based on heuristic methods and the inclusion of the decision-maker (experts) as a main component of the decision-making task.

Effective instruments are needed to build complex decision-making procedures and to evaluate a wide range of alternatives. The present paper preferred a modern multi-objective choice method – TOPSIS, modified to suit the conditions of the solved problem.

3.2. **TOPSIS method.** The main idea of the TOPSIS method is that the most preferable alternative should have the shortest distance from the ideal solution and the longest geometric among all alternatives from the inadmissible solution [14]. Here, the best (optimal) solution is a vector that contains maximal values by each criterion for all alternatives, while the inadmissible (worst) solution is a vector that contains minimal values by each criterion for all alternatives. Based on the essence of the TOPSIS method, the use of the latter is efficient in solving tasks of fuzzy multi-objective optimization, which constitute the mathematical basis of decision-making support in human resource management tasks. In the decision-making theory, multi-objective optimization is understood as the selection of the best solution among the possible alternatives [31].

The solution of optimization tasks with the use of TOPSIS assumes the need for translating the values of qualitative linguistic variables that express the level of satisfaction of the criteria by this or than alternative into fuzzy numbers.

A fuzzy number is a fuzzy subset of a universal set of real numbers, which has a normal or convex membership function, for which there exists such a carrier value, where the membership function is 1, while the membership function decreases during leftwards or rightwards deviation [17]. According to [43], fuzzy expert judgments that were formulated in the natural language can be described by fuzzy triangular and fuzzy trapezoidal numbers. This paper, taking into consideration the need for ensuring the robustness of criteria to the confidence interval boundaries, uses a fuzzy trapezoidal number.

Operations on fuzzy numbers are introduced by means of operations on membership functions, based on the segmental principle [45].

When using the TOPSIS method, one should consider certain operations on fuzzy numbers.

Given two fuzzy trapezoidal numbers $\overline{n} = (n_1, n_2,\ n_3,\ n_4)$ and $\overline{m} = (m_1,\ m_2,\ m_3,\ m_4)$, the following are the operations of summation, difference, and multiplication of said numbers:

$$\begin{aligned}
&\overline{n} \oplus \overline{m} = [n_1 + m_1,\ n_2 + m_2,\ n_3 + m_3,\ n_4 + m_4]\,,\\
&\overline{n} - \overline{m} = [n_1 - m_4,\ n_2 - m_3,\ n_3 - m_2,\ n_4 - m_1]\\
&\overline{n} \otimes \overline{m} \cong [n_1 m_1,\ n_2 m_2,\ n_3 m_3,\ n_4 m_4]\\
&\overline{n} \otimes r = [n_1 r,\ n_2 r,\ n_3 r,\ n_4 r]\\
&\overline{n} \div \overline{m} \cong \begin{bmatrix} \min(n_1 \div m_1,\ n_1 \div m_4,\ n_4 \div m_1,\ n_4 \div m_4),\ n_2 \div m_2,\ n_3 \div m_3, \\ \max(n_1 \div m_1,\ n_1 \div m_4,\ n_4 \div m_1,\ n_4 \div m_4) \end{bmatrix}\\
&\max(\overline{n}, \overline{m}) = (\max(n_1,\ m_1),\ \max(n_2,\ m_2),\ \max(n_3, m_3),\ \max(n_4, m_4)\\
&\min(\overline{n}, \overline{m}) = (\min(n_1,\ m_1),\ \min(n_2,\ m_2),\ \min(n_3, m_3),\ \min(n_4, m_4).
\end{aligned} \tag{1}$$

The distance between two fuzzy trapezoidal numbers is determined by the following expression [6, 16]:

$$d_c(\overline{n},\ \overline{m}) = \sqrt{\frac{1}{4}((n_1 - m_1)^2 + (n_2 - m_2)^2 + (n_3 - m_3)^2 + (n_4 - m_4)^2)} \tag{2}$$

If $\overline{n} = \overline{m}$, i.e. $\overline{n}$ and $\overline{m}$ are equal, then $d_c(\overline{n},\ \overline{m}) = 0$.

In order to implement this method, one should handle linguistic variables and their values that express verbal ranking scales for measuring attributes. Here, the levels are arranged in the order of ascension of these attributes' intensity. In this case, the number of linguistic variables' values (ranks) is seven. Fig.1 shows a graphical representation of the transformation of linguistic values into numeric equivalents.

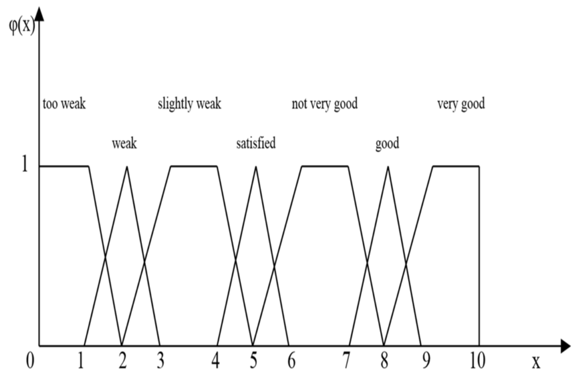

Figure 1. Transformation of linguistic values into fuzzy trapezoidal numbers.

Table 1 shows the 7-level values of the linguistic variable and respective fuzzy trapezoidal numbers.

Table 1. Linguistic values and their respective fuzzy trapezoidal numbers.

Linguistic values	Fuzzy trapezoidal numbers
too weak	(0,0,1,2)
weak	(1,2,2,3)
slightly weak	(2,3,4,5)
satisfactory	(4,5,5,6)
not very good	(5,6,7,8)
good	(7,8,8,9)
very good	(8,9,10,10)

According to Table 1, a numeric equivalent can be found for each linguistic variable value.

3.3. **TOPSIS-based algorithm of multi-objective optimization in HRM tasks.** The goal of the task is to rank alternatives, based on the evaluations of experts, taking into consideration the competence of the latter. The solution of the task assumes the performance of the following sequence of actions:

Step 1. In order to perform TOPSIS-based multi-objective optimization of HRM tasks, one should first dispose of the hierarchal structure of criteria (Fig. 2). For this purpose, based on Saaty's AHP, by relative importance coefficients of criteria $\left\{K_j,\ j=\overline{1,\ m}\right\}$ and partial criteria $\left\{k_{jt},\ t=\overline{1,\ s_j}\right\}$, weights are determined [34, 35], with which the latter will enter the calculation of the K integral criterion. In a formalized form, w_{jt}^K – the weight of the k_{jt} partial criterion in the calculation of the integral criterion $K=\left\{k_j,\ j=\overline{1,\ m}\right\}$, i.e. $w_{jt}^K = w_{jt}\cdot w_j$, is determined by the multiplication w_j, where $\sum\limits_{j=1}^{m} w_j \ = 1$, and w_{jt}, where $\sum\limits_{t=1}^{s_j} w_{jt} \ = 1$.

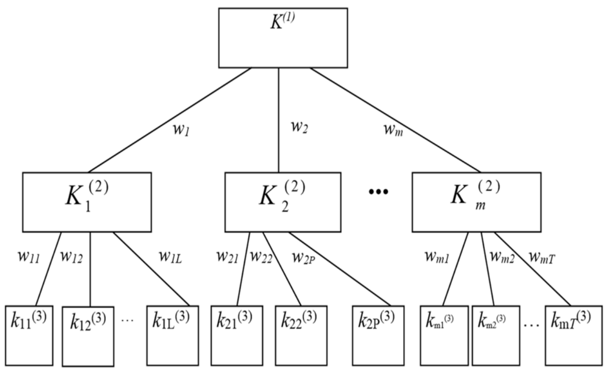

Figure 2. Hierarchal structure of choice criteria that characterize alternatives.

As a result, the two-level hierarchal structure of choice criteria $K = \{K_j,\ j = \overline{1,\ m}\}$ that characterize alternatives comes down to the calculation of the integral criterion that includes the weights of partial criteria $\{k_{jt},\ t = \overline{1,\ s_j}\}$, which allows disposing of the hierarchal structure (Fig.3).

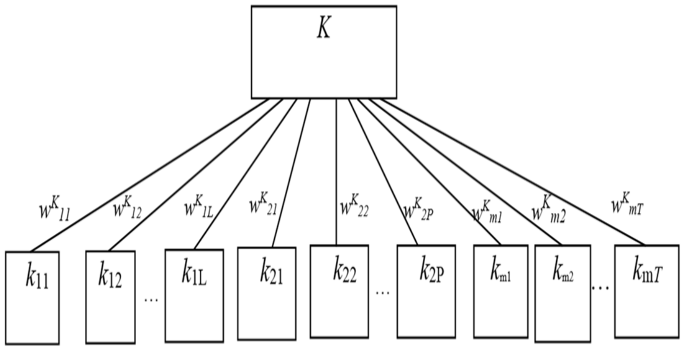

Figure 3. Reduction of the K-criterion hierarchal structure to the integral vector of partial criteria.

During subsequent steps, all partial criteria are united into a single G set, with a view to simplifying indexes.

$$G = \{k_{jt},\ j = \overline{1, m},\ t = \overline{1,\ s_j}\} = \{k_z,\ z = \overline{1, Z}\}, z = s_{j-1} + t,\ j = \overline{1, m},\ t = \overline{1,\ s_j},\ s_0 = 0.$$

Here, Z is the overall number of partial criteria that characterize alternatives, i.e. $Z = \sum\limits_{j=1}^{m} s_j$.

In this case, $w_z = w^K_{jt}$.

Step 2. The level of membership (relation) of alternatives on partial criteria is evaluated by linguistic values (see Table 1) and expressed by trapezoidal numbers $R^l = (r^l_{iz}) = (a^l_{iz},\ b^l_{iz},\ c^l_{iz},\ d^l_{iz})$. For example, if the level of satisfaction (membership) by alternative x_i of partial criterion k_z is evaluated as “good” by expert l, it is expressed as $r^l_{iz} = (7,\ 8,\ 8,\ 9)$, while if the expert gives a “very good” evaluation, then $r^l_{iz} = (8,\ 9,\ 10,\ 10)$, etc. The expert evaluation of alternatives' membership on partial criteria results in the following matrix:

$$R^l = \left[r^l_{iz}\right] \Leftrightarrow \left\{a^l_{iz},\ b^l_{iz},\ c^l_{iz},\ d^l_{iz}\right\},\ l = \overline{1, g}.$$

Step 3. This step assumes the pre-estimation of experts' competence coefficient $v_l,\ l = \overline{1, g}$. For this purpose, the authors applied the modified method that integrates into the algorithm pre-estimated coefficients of competence of experts, who participate in the evaluation of alternatives.

The matrix $R^{v_l} = [r^{v_l}_{iz}],\ l = \overline{1,\ g} \Leftrightarrow \{a^{v_l}_{iz},\ b^{v_l}_{iz},\ c^{v_l}_{iz},\ d^{v_l}_{iz}\},\ l = \overline{1, g}$ is formed, taking into account the experts' competence coefficient $v_l,\ l = \overline{1, g}$. The elements of this matrix are trapezoidal numbers that express the level of satisfaction by alternative x_i of partial criteria k_z, taking into account the experts' competence. The elements are calculated as follows:

$$a^{v_l}_{iz} = a^l_{iz} \cdot v_l; b^{v_t}_{iz} = b^l_{iz} \cdot v_l; c^{v_l}_{iz} = c^l_{iz} \cdot v_l; d^{v_l}_{iz} = d^l_{iz} \cdot v_l \tag{3}$$

Step 4. This step determines the single aggregated matrix:

$$R^{v_l} = [r^{v_l}_{iz}] \Leftrightarrow \{a^{v_l}_{iz},\ b^{v_l}_{iz},\ c^{v_l}_{iz},\ d^{v_l}_{iz}\},\ l = \overline{1, g} \Rightarrow R_{iz} = [r_{iz}] \Leftrightarrow \{a_{iz},\ b_{iz},\ c_{iz},\ d_{iz}\}.$$

The elements of this matrix are determined as follows:

$$\begin{aligned} a_{iz} &= \left\{\min\ a_{iz}^{v_l},\ l=\overline{1,g}\right\};\\ b_{iz} &= \tfrac{1}{g}\sum_{l=1}^{g} b_{iz}^{v_l};\\ c_{iz} &= \tfrac{1}{g}\sum_{l=1}^{g} c_{iz}^{v_l};\\ d_{iz} &= \left\{\max\ d_{iz}^{v_l},\ l=\overline{1,g}\right\}. \end{aligned} \tag{4}$$

Step 5. The elements of matrix $R_{iz} = [r_{iz}] \Leftrightarrow \{a_{iz},\ b_{iz},\ c_{iz},\ d_{iz}\}$ are multiplied by the weights of partial criteria. This operation builds the weighed fuzzy matrix $R_{iz}^{w} = [r_{iz}^{w}] \Leftrightarrow \{a_{iz}^{w},\ b_{iz}^{w},\ c_{iz}^{w},\ d_{iz}^{w}\}$. Here:

$$\begin{aligned} a_{iz}^{w} &= a_{iz}\cdot w_z;\\ b_{iz}^{w} &= b_{iz}\cdot w_z;\\ c_{iz}^{w} &= c_{iz}\cdot w_z;\\ d_{iz}^{w} &= d_{iz}\cdot w_z. \end{aligned} \tag{5}$$

Step 6. The obtained matrix is normalized. For this purpose, the *Hsu* and *Cehn* method [13] is used:

$$R_{iz}^{N} = \left[r_{iz}^{N}\right] \Leftrightarrow \left[\frac{r_{iz}^{w}}{\max\limits_{i} r_{iz}^{w}}\right] = \left[\frac{r_{iz}^{w}}{r_{z}^{w+}}\right] = \left\{a_{iz}^{N},\ b_{iz}^{N},\ c_{iz}^{N},\ d_{iz}^{N}\right\}. \tag{6}$$

Here:

$$\begin{aligned} a_{iz}^{N} &= \min(a_{iz}^{w}\div\max_i a_{iz}^{w},\ a_{iz}^{w}\div\max_i d_{iz}^{w},\ d_{iz}^{w}\div\max_i a_{iz}^{w},\ d_{iz}^{w}\div\max_i d_{iz}^{w});\\ b_{iz}^{N} &= b_{iz}^{w}\div\max_i b_{iz}^{w};\\ c_{iz}^{N} &= c_{iz}^{w}\div\max_i c_{iz}^{w};\\ d_{iz}^{N} &= \max(a_{iz}^{w}\div\max_i a_{iz}^{w},\ a_{iz}^{w}\div\max_i d_{iz}^{w},\ d_{iz}^{w}\div\max_i a_{iz}^{w},\ d_{iz}^{w}\div\max_i d_{iz}^{w}). \end{aligned}$$

Step 7. Based on the weighed values, the positive ideal (optimal) solution (PIS) X^{*} is determined. For this purpose, for each $k_z,\ z=\overline{1,\ Z}$

$$r_z^{+} = \max_i r_{iz}^{N} = \left\{\max_i a_{iz}^{N},\ \max_i b_{iz}^{N},\ \max_i c_{iz}^{N},\ \max_i d_{iz}^{N}\right\} = \left\{a_z^{+},\ b_z^{+},\ c_z^{+},\ d_z^{+}\right\} \tag{7}$$

is selected, and the

$$X^{+} = \left[r_z^{+}\right] = (r_1^{+},\ r_2^{+},...,\ r_Z^{+}) = (\max_i r_{i1}^{N},\ \max_i r_{i2}^{N},\ ...,\ \max_i r_{iZ}^{N}). \tag{8}$$

matrix is formed.

Step 8. The negative (worst) ideal value (NIS) X^{-} is calculated. For this purpose, for each $k_z,\ z=\overline{1,\ Z}$

$$r_z^{-} = \min_i r_{iz}^{N} = \left\{\min_i a_{iz}^{N},\ \min_i b_{iz}^{N},\ \min_i c_{iz}^{N},\ \min_i d_{iz}^{N}\right\} = \left\{a_z^{-},\ b_z^{-},\ c_z^{-},\ d_z^{-}\right\} \tag{9}$$

is selected, and the following matrix is formed:

$$X^{-} = \left[r_z^{-}\right] = (r_1^{-},\ r_2^{-},...,\ r_Z^{-}) = (\min_i\ r_{i1}^{N},\ \min_i r_{i2}^{N},...,\ \min_i r_{iZ}^{N}). \tag{10}$$

Step 9. The distance of alternatives from PIS are calculated by formula (2) for the individual values of each partial criterion:

$$D_z^+(x_i,\ X^+) = \sqrt{\frac{1}{4}((a_{iz}^N - a_z^+)^2 + (b_{iz}^N - b_z^+)^2 + (c_{iz}^N - c_z^+)^2 + (d_{iz}^N - d_z^+)^2)}. \tag{11}$$

Vector $[D^*] = [D_1^*, ..., D_Z^*]$ is formed, based on the obtained results.

Step 10. The distance of alternatives from NIS are calculated for the individual values of each partial criterion

$$D_z^-(x_i,\ X^-) = \sqrt{\frac{1}{4}((a_{iz}^N - a_z^-)^2 + (b_{iz}^N - b_z^-)^2 + (c_{iz}^N - c_z^-)^2 + (d_{iz}^N - d_z^-)^2)}. \tag{12}$$

Vector $[D^-] = \left[D_1^-, ..., D_Z^-\right]$ is formed, based on the obtained results.

Step 11. The distance of each alternative from PIS is determined:

$$D^*(x_i) = \sqrt{\sum_{z=1}^{Z}(D_z^+(x_i,\ X^+))^2}. \tag{13}$$

Step 12. The distance of each alternative from NIS is determined:

$$D^-(x_i) = \sqrt{\sum_{z=1}^{Z}(D_z^-(x_i,\ X^-))^2}. \tag{14}$$

Step 13. The integral indicator (proximity coefficient) is calculated for each compared alternative, as the correlation between its calculated distance from the negative ideal solution, and the sum of distances between the best and the worst solutions:

$$\begin{aligned} D(x_i) &= D^+(x_i) + D^-(x_i) \\ \varphi(x_i) &= \tfrac{D^-(x_i)}{D(x_i)}. \end{aligned} \tag{15}$$

The value of the proximity coefficient $\varphi(x_i)$ allows ranking alternatives. For example, the closer the value of the proximity coefficient $\varphi(x_i)$ to 1, the more preferable the compared alternative.

3.4. **Application of the suggested method for solving personnel selection and recruitment tasks.** The suggested instrumental approach was tested during the solution of tasks of selecting and recruiting with a view to evaluating candidates. Experiments were conducted with a view to evaluating the applicants for the vacancy at the State Oil Company of the Azerbaijan Republic (SOCAR) Human Resource Management Department. For this purpose, the following actions were taken:

1. A criteria system (Tab.2.) was formed with the participation of four experts, with a view to recruiting personnel to the HRM Department. The coefficients of criteria's relative importance were calculated, based on pairwise comparison [35]. The task of detecting contradictions in expert evaluations [15, 35] was also considered. The obtained results helped determine the coefficients of criteria's relative importance, and the weights of partial criteria, with which the latter will enter the calculation of the K integral criterion.

Table 2. Coefficients of relative importance of criteria and partial criteria, weight of partial criteria in K.

criteria	criteria relative importance coefficients	partial criteria	partial criteria relative importance coefficients	partial criteria weight coefficients
K_1 Professional (education, knowledge, professional skills, abilities, etc.)	0.11	k_{11} conformity of the education to the job requirements	0.54	0.06
		k_{12} scientific and research abilities	0.46	0.05
K_2 Motivational	0.08	k_{21} sense of purpose	0.47	0.04
		k_{22} result orientation	0.53	0.04
K_3 Business	0.4	k_{31} diligence	0.2	0.08
		k_{32} creativity	0.22	0.13
		k_{33} initiative	0.26	0.10
		k_{34} self-sufficiency	0.32	0.09
K_4 Personal	0.1	k_{41} trainability	0.63	0.06
		k_{42} can-do attitude	0.37	0.04
K_5 Individual-psychological and health	0.31	k_{51} physical health	0.35	0.11
		k_{52} psychological resilience	0.65	0.20

2. The obtained integral indicator (the proximity coefficient of compared alternatives) $\varphi(x_i)$ was expressed by a certain value (on the [0,1] interval) of the probability of recruitment for each candidate x_i. The values of this variable allow making the final decision regarding each alternative candidate. During the conduction of recruitment experiments, experts formulated the following variants of possible final decisions:

(1) If $\varphi(x_i) \in [0, 0.25)$, the candidate obviously does not conform to the requirements of the position, i.e. the candidate is turned down;
(2) If $\varphi(x_i) \in [0.25, 0.45)$, the candidate weakly conforms to the requirements of the position, therefore, his recruitment poses high risk.
(3) If $\varphi(x_i) \in [0.45, 0.65)$, the candidate partially (to a certain extent) conforms to the requirements of the position. The recruitment of the candidate poses low risk, which can be compensated for by high indicators in other competencies during work;
(4) If $\varphi(x_i) \in [0.65, 0.85)$, the candidate conforms to the requirements of the position, while certain indicators can be easily improved during adaptation;
(5) If $\varphi(x_i) \in [0.85, 1)$, the candidate fully conforms to the requirements of the position.

3. The coefficients of the competence of experts, who participate in the evaluation of candidates, were calculated by pairwise comparison [15, 35], based on the linguistic expression of "slight superiority of expert 1 and expert 4 over expert 2 and expert 3": *v1=0,375, v2=0,125, v3=0,125, v4=0,375.*

4. With the participation of four experts, based on seven-level linguistic variables, the authors evaluated the level of satisfaction (membership) of 12 partial criteria by three candidates for the job, who passed all necessary stages of selection.

5. A $3x12x4$ generalized matrix of fuzzy trapezoidal numbers (Tab.3.) was built, based on the evaluations of four experts.

Table 3. Matrix of fuzzy trapezoidal numbers that reflect the membership of alternatives in partial criteria.

Partial criteria (k_z)	Alternatives	Experts			
		Expert 1	Expert 2	Expert 3	Expert 4
k_1	x_1	(7,8,8,9)	(7,8,8,9)	(7,8,8,9)	(7,8,8,9)
	x_2	(7,8,8,9)	(8,9,10,10)	(4,5,5,6)	(5,6,7,8)
	x_3	(7,8,8,9)	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)
k_2	x_1	(8,9,10,10)	(8,9,10,10)	(7,8,8,9)	(8,9,10,10)
	x_2	(5,6,7,8)	(7,8,8,9)	(8,9,10,10)	(5,6,7,8)
	x_3	(7,8,8,9)	(8,9,10,10)	(8,9,10,10)	(4,5,5,6)
k_3	x_1	(7,8,8,9)	(7,8,8,9)	(4,5,5,6)	(5,6,7,8)
	x_2	(7,8,8,9)	(8,9,10,10)	(8,9,10,10)	(7,8,8,9)
	x_3	(8,9,10,10)	(7,8,8,9)	(7,8,8,9)	(8,9,10,10)
k_4	x_1	(5,6,7,8)	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)
	x_2	(8,9,10,10)	(8,9,10,10)	(8,9,10,10)	(8,9,10,10)
	x_3	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)	(7,8,8,9)
k_5	x_1	(7,8,8,9)	(5,6,7,8)	(8,9,10,10)	(7,8,8,9)
	x_2	(8,9,10,10)	(7,8,8,9)	(7,8,8,9)	(8,9,10,10)
	x_3	(7,8,8,9)	(8,9,10,10)	(8,9,10,10)	(7,8,8,9)
k_6	x_1	(7,8,8,9)	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)
	x_2	(8,9,10,10)	(5,6,7,8)	(7,8,8,9	(8,9,10,10)
	x_3	(5,6,7,8)	(7,8,8,9)	(8,9,10,10)	(5,6,7,8)
k_7	x_1	(8,9,10,10)	(8,9,10,10)	(8,9,10,10)	(8,9,10,10)
	x_2	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)	(7,8,8,9)
	x_3	(7,8,8,9)	(7,8,8,9)	(4,5,5,6)	(5,6,7,8)
k_8	x_1	(7,8,8,9)	(8,9,10,10)	(8,9,10,10)	(4,5,5,6)
	x_2	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)	(7,8,8,9)
	x_3	(5,6,7,8)	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)
k_9	x_1	(8,9,10,10)	(7,8,8,9)	(7,8,8,9)	(8,9,10,10)
	x_2	(7,8,8,9)	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)
	x_3	(8,9,10,10)	(5,6,7,8)	(7,8,8,9)	(8,9,10,10)
k_{10}	x_1	(5,6,7,8)	(7,8,8,9)	(8,9,10,10)	(5,6,7,8)
	x_2	(7,8,8,9)	(8,9,10,10)	(8,9,10,10)	(4,5,5,6)
	x_3	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)	(7,8,8,9)
k_{11}	x_1	(8,9,10,10)	(7,8,8,9)	(7,8,8,9)	(7,8,8,9)
	x_2	(5,6,7,8)	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)
	x_3	(8,9,10,10)	(8,9,10,10)	(8,9,10,10)	(8,9,10,10)
k_{12}	x_1	(7,8,8,9)	(8,9,10,10)	(7,8,8,9)	(7,8,8,9)
	x_2	(7,8,8,9)	(7,8,8,9)	(4,5,5,6)	(5,6,7,8)
	x_3	(7,8,8,9)	(8,9,10,10)	(8,9,10,10)	(7,8,8,9)

6. Taking into account the competence of experts based on the formula (3), the matrix of trapezoidal fuzzy numbers was built and aggregated in accordance with formula (4) trapezoidal fuzzy numbers were defined (Tab.4).

7. The elements of the matrix of aggregated fuzzy trapezoidal numbers were multiplied by the weights of partial criteria according to formula (5), and the results were normalized (Tab.4).

Table 4. Elements of the normalized decision-making matrix.

Partial criteria	Alternatives	Partial criteria weight coefficients	Aggregated trapezoidal fuzzy number	elements of the weighed fuzzy matrix (x100)	$\max_i r_{iz}^w$	elements of the normalized decision-making matrix
k_{11}	X_1	0.06	(0.875,2,2,3.375)	(5.25, 12, 12, 20.25)	(5.25, 12.186, 12.36, 20.25)	(0.259, 0.985, 0.971, 3.857)
	X_2		(0.5,1.715, 1.88, 3.375)	(3, 10.29, 11.28, 20.25)		(0.148, 0.844, 0.913, 3.857)
	X_3		(0.875,2.031,2.06, 3.375)	(5.25, 12.186, 12.36, 20.25		(0.259, 1,1, 3.857)
k_{12}	X_1	0.05	(0.875,2.219,2.44,3.75)	(4.375, 11.08, 12.2, 18.75)	(5.5, 11.08, 12.2, 18.75)	(0.233, 1, 1, 4.286)
	X_2		(0.875,1.586,1.926,3.375)	(4.375, 7.93, 9.63, 16.875)		(0.233, 0.716, 0.789, 3.068)
	X_3		(1.1,1.781,1.846,3.375)	(5.5, 8.905, 9.23, 16.875)		(0.293, 0.804, 0.757, 3.068)
k_{21}	X_1	0.04	(0.5, 1.684, 1.862, 3.375)	2, 6.736, 7.448, 13.5)	(4, 8.752, 9.54, 15)	(0.133, 0.770, 0.781, 3.375)
	X_2		(1, 2.062, 2.125, 3,375)	(4, 8.248, 8.5, 13.5)		(0.266, 0.942, 0.891, 3.375)
	X_3		0.875, 2.188, 2.385, 3.75	(3.5, 8.752, 9.54, 15)		(0.233, 1, 1, 3.75)
k_{22}	X_1	0.04	(0.875, 1.809, 1.969, 3.375)	(3.5, 7.236, 7.876, 13.5)	(4, 9, 10, 15)	(0.233, 0.804, 0.788, 3.375)
	X_2		(1, 2.25, 2.5, 3.75)	(4, 9, 10, 15)		(0.266, 1, 1, 3.75)
	X_3		(0.875, 2.031, 2.062, 3.375)	(3.5, 8.124, 8,248, 13.51)		(0.233, 0.903, 0.825, 3.375)
k_{31}	X_1	0.08	(0.5, 1.68, 1.835, 3.375)	(4, 13.44, 14.68, 27)	(8, 17.52, 19.04, 30)	(0.133, 0.767, 0.771, 3.375)
	X_2		(0.875, 2.19, 2.38,3.75)	(7, 17.52, 19.04, 30)		(0.233, 1, 1, 3.75)
	X_3		(1, 2.03, 2.08, 3.375)	(8, 16.24, 16.64, 27)		(0.266, 0.927, 0.874, 3.375)
k_{32}	X_1	0.03	(0.875, 2.03, 2.06, 3.375)	(2.625, 6.09, 6.18, 10.125)	(2.625, 6.657, 6.975, 11.25)	(0.233, 0.915, 0.886, 3.857)
	X_2		(0.625, 2.219, 2.325, 3.75)	(1.825, 6.657, 6.975, 11.25		(0.162, 1, 1, 4.286)
	X_3		(0.875, 1.565, 1.9, 3.375)	(2.625, 4.695, 5.7, 10.125)		(0.233, 0.705, 0.817, 3.857)
k_{33}	X_1	0.10	(1, 2.25, 2.5, 3.75)	(10, 22.5, 25, 37.5)	(10, 22.5, 25, 37.5)	(0.266, 1, 1, 3.75)
	X_2		(0.875, 2.031, 2.06, 3.375)	(8.75, 20.31, 20.6, 33.75)		(0.233, 0.903, 0.824, 3.375)
	X_3		(0.5, 1.684, 1.86, 3.375)	(5, 16.84, 18.6, 33.75)		(0.133, 0.748, 0.744, 3.375)
k_{34}	X_1	0.09	(1, 1.781, 1.9, 3.375)	(9, 16.029, 17.1, 30.375)	(9, 18.279, 18.54, 30.375)	(0.296, 0.877, 0.922, 3.375)
	X_2		(0.875, 2.031, 2.062, 3.375)	(7.875, 18.279, 18.54, 30.375)		(0.259, 1, 1, 3.375)
	X_3		(0.875, 1.836, 1.969, 3.375)	(7.875, 16.524, 17.721, 30.375)		(0.259, 0.904, 0.956, 3.375)
k_{41}	X_1	0.06	(0.875, 2.188, 2.385, 3.75)	(5.25, 13.128, 14.31, 22.51	(5.25, 13.128, 14.31, 22.51)	(0.233, 1, 1, 4.286)
	X_2		(0.875, 2.031, 2.062, 3.375)	(5.25, 12.186, 12.372, 20.2		(0.233, 0.928, 0.865, 3.857)
	X_3		(0.625, 2.125, 2.343, 3.75)	(3.75, 12.75, 14.058, 22.51		(0.166, 0.971, 0.982, 4.286)
k_{42}	X_1	0.04	(0.875, 1.586, 1.875, 3.375)	(3.5, 6.34, 7.5, 13.51)	(4, 8.124, 8.248, 13.51)	(0.259, 0.780, 0.909, 3.375)
	X_2		(1, 1.781, 1.843, 3.375)	(4, 7.124, 7.372, 13.51)		(0.296, 0.877, 0.894, 3.375)
	X_3		(0.875, 2.031, 2.062, 3.375)	(3.5, 8.124, 8.248, 13.51)		(0.259, 1, 1, 3.375)

k_{s1}	x_1	0.11	(0.875, 2.093, 2.187, 3.75)	(9.625, 23.023, 24.057, 41.25)	(11, 24.75, 27.5, 41.25)	(0.233, 0.930, 0.875, 3.75)
	x_2		(0.875,1.808, 1.968, 3.375)	(9.625, 19.888, 21.648, 37.125)		(0.233, 0.804, 0.787, 3.375)
	x_3		(1, 2.25, 2.5, 3.75)	(11, 24.75, 27.5, 41.25)		(0.267, 1, 1, 3.75)
k_{s2}	x_1	0.20	(0.875, 2.031, 2.063, 3.375)	(17.5, 40.62, 41.26, 67.5)	(20, 41.24, 42.5, 67.5)	(0.259, 0.985, 0.971, 3.375)
	x_2		(0.5, 1.683, 1.812, 3.375)	(10, 33.66, 36.24, 67.5)		(0.148, 0.816, 0.853, 3.375)
	x_3		(1, 2.062, 2.125, 3.375)	(20, 41.24, 42.5, 67.5)		(0.296, 1, 1, 3.375)

8. The integral matrix of fuzzy positive (best) ideal solutions and fuzzy negative (worst) ideal solutions was formed in accordance with expressions (7)-(10). The matrix is presented in Tab.5.

9. The results of the calculations of distances between alternatives and PIS were calculated, based on formula (11) for the value of each partial criterion. The results are presented in Tab.5.

10. The results of the calculations of distances between alternatives and NIS were calculated, based on formula (12) for the value of each partial criterion. The results are presented in Tab 5.

Table 5. Distance of alternatives from NIS and PIS by values of each partial criterion.

Partial criteria	X^*	X^-	$D(x_1X^*)$	$D(x_2X^*)$	$D(x_3X^*)$	$D(x_1X^-)$	$D(x_2X^-)$	$D(x_3X^-)$
k_1	(0.259,1,1,3.857)	(0.148, 0.844,0.913, 3.857)	0.01612	0.10515	0.000	0.09388	0	0.10516
k_2	(0.293,1,1,4.286)	(0.233, 0.716, 0.757, 3.068)	0.03	0.58711	0.58059	0.19216	0.016	0.05325
k_3	(0.266,1,1,3.75)	(0.133, 0.770, 0.781, 3.068)	0.0637	0.03889	0.01643	0	0.13669	0.25568
k_4	(0.266,1,1,3.75)	(0.233, 0.804, 0.788, 3.375)	0.23717	0.000	0.21312	0	0.23721	0.19481
k_5	(0.266,1,1,3.75)	(0.133, 0.767, 0.771, 3.375)	0.25741	0.016	0.20114	0	0.25365	0.11608
k_6	(0.233,1,1,4.286)	(0.162, 0.705, 0.771, 3.857)	0.47948	0.0355	0.27593	0.11609	0.147	0.03550
k_7	(0.266,1,1,3.75)	(0.133, 0.748, 0.744, 3.375)	0.000	0.21342	0.49862	0.24918	0.10053	0
k_8	(0.296,1,1,3.375)	(0.259, 0.877, 0.922, 3.375)	0.20115	0.18755	0.29292	0.02	0.07283	0.02147
k_9	(0.233,1,1,4.286)	(0.166, 0.928, 0.865, 3.857)	0.000	0.15085	0.02345	0.23018	0.0335	0.22337
k_{10}	0.296,1,1,3.375)	(0.259, 0.780, 0.894, 3.375)	0.12554	0.07642	0.01	0.00748	0.05190	0.11904
k_{11}	(0.266,1,1,3.75)	(0.233, 0.804, 0.787, 3.375)	0.07071	0.221	0.000	0.07684	0	0.23747
k_{12}	(0.296,1,1,3.375)	(0.148, 0.816, 0.853, 3.375)	0.02468	0.13943	0.000	0.11811	0	0.13908

11. The distances from each alternative to PIS and DIS were calculated in accordance with formulas (13), (14), respectively. Formula (15) was used to calculate the values of the integral indicator, which expresses the proximity of each compared alternative to the ideal solution. The ranks of each alternative were determined in accordance with the results (Table 6).

Table 6. Distance of compared alternatives from PIS and NIS, the coefficient of their proximity to the ideal solution and respective ranks.

Alternatives	X^*	X^-	$X^* + X^-$	$\varphi_K(x_i)$	**Ranks**	**Solution**
x_1	0.46847	0.44805	0,91652	0.48886	1	reception of the candidate is associated with a small risk, which can be compensated by high performance in other competencies
x_2	0.80634	0.41503	1,22137	0,33981	3	reception the candidate is associated with a great risk
x_3	0.91326	0.52174	1,43500	0.36358	2	reception of the candidate is associated with a great risk

According to the obtained results, the best (optimal) solution in this case is the x_3 alternative, i.e. candidate x_3.

4. Results

Mathematically reasonable and relatively simple algorithm for calculating the integral estimates, provided by TOPSIS method, ensures its applicability in a wide range of practical problems. The TOPSIS based of multi-criteria fuzzy group decision making method have been used to solve the problem of choosing mid-level managers in the Greek IT company [19], for the selection of human resources at a large Greek bank [33], to improve the process of selecting and hiring staff in Iran Khodro Company [32], to select musicians in the rock band [46], for hiring IT professionals [29].

The authors proposed the modification, which amounts to the introduction of scientific expertise in algorithm factors involved in the process of evaluating alternatives, and the integrated use of previously entered in the works [19, 29, 32, 33, 46] certain TOPSIS modifications (the group decision making, the hierarchical structure of the criteria, the relative importance of the criteria), extended the possibilities of the method, ensuring great value for decision-making in HRM problems.

The carried out step by step procedure of the modified TOPSIS method implementation, on the example of the problem of candidates selection for the position in HR Department of SOCAR, has once again demonstrated the practical effectiveness of the proposed approach for: a) more objective evaluation of the level of professional qualifications and personal characteristics of applicants, b) the support of managers in the process of informed decision-making for the selection of personnel.

5. Discussion

Since SOCAR Department of HRM recruits the applicants to the position in a real situation based on the scoring system for evaluating alternatives, we also conducted a scoring of alternatives for comparison purposes. Based on the linguistic assessments of experts, a total score for each alternative is calculated (Tab. 4). To assess the alternatives based on the point system, linguistic values "very good" correspond to 10 points, "good" - 8 points, "not very good" - 6 points and "satisfactory" - 4 points.

For example, according to the linguistic assessments of four experts by 12 indicators, the followings were obtained for the alternative x_1: 17 - "very good", 24 - "good", 5 - "not very good" and 2 - "satisfactory" (Tab. 4). Based on the point system, the total scoring evaluation (TSE) for x_1 equals to 400 and is calculated according to the expression as follows:

$$x_1^{ss} = 17 \times 10 + 24 \times 8 + 5 \times 6 + 2 \times 4 = 400.$$

The total scoring evaluation for x_2 and x_3 is determined in a similar way.

The choice of an ideal alternative based on the point system is determined by dividing the total score of the alternatives by the maximum score value (ideal solution $x^{is} = 480$, i.e. $4 \times 12 \times 10$, where 4 is the number of experts, 12 - number of particular criteria, and 10 - maximum score). For example, for alternative x_1, the alternatives referring to an ideal solution are determined as follows: $\varphi(x_1^{ss}) = x_1^{ss}/x^{is} = 400/480 = 0,833$.

As it is seen from Tab.7, the best alternative is x_3, which "scored" 406 points, in order of preference, followed by the alternative x_1 and the least preferable alternative x_2. Hence, according to the degree of proximity to the ideal solution, an appropriate conclusion is made for each alternative.

Table 7. Results of scoring of alternatives taking into account linguistic assessments from Table 4.

alternatives	very good	good	not very good	satisf.	final score x_i^{ss}	membership of alternatives on the ideal solution $\varphi(x_i^{ss})$	**Solution**
x_1	17	24	5	2	400	0.833	the candidate meets all the requirements of the workplace
x_2	18	21	6	3	396	0.825	the candidate meets all the requirements of the workplace
x_3	19	22	6	1	406	0.846	the candidate meets all the requirements of the workplace

Table 8 presents the results of three approaches to assessing and prioritizing alternatives in terms of the degree of proximity to an ideal solution, which allows their comparative analysis.

Table 8. Results of the prioritization of alternatives in accordance with the three approaches.

Alternatives	The results obtained based on the method proposed in the article		Results of 10-point score for the evaluation of alternatives	
	Ranks	Decision making	Ranks	Decision making
x_1	1	reception of the candidate is associated with a small risk, which can be compensated by high performance in other competencies	2	the candidate meets all the requirements of the workplace
x_2	3	reception the candidate is associated with a great risk	3	the candidate meets all the requirements of the workplace
x_3	2	reception of the candidate is associated with a great risk	1	the candidate meets all the requirements of the workplace

Comparison of the results of calculations by two methods shows an obvious inconsistency between the latter ones. Moreover, candidates, whose admission to work according to the proposed method is associated with a high risk, are referred to the category of the most preferable candidates when scoring the alternatives. Thus, the proposed method is more sensitive when selecting the best alternative, i.e. it offers the choice of the most appropriate alternative, that is the best one among the best.

Thus, the results of approbation show a sufficient sensitivity of the proposed method when selecting the best alternative among the best, whereas the scoring method actually does not provide differentiating several alternatives with highest priority in terms of proximity to the ideal one.

6. Conclusion

The suggested methodological approach to the solution of HRM task with the use of TOPSIS-based multi-objective optimization allows improving the adequacy of made decisions by means of prioritization by the proximity to the ideal solution, ensures the objectiveness and transparency of managerial decisions, and provides opportunities for expanding the applicability of multi-objective optimization methods.

The advantages of the suggested approach to multi-objective optimization, based on a modified TOPSIS method, with a view to supporting decision-making in human resource management, are as follows:

(1) the lack of necessity of compiling a fuzzy rules base;
(2) the mathematical validity and relative simplicity of calculating integral indicators, which allow ranking alternative decisions, conducting further analysis, and selecting the final variant of the decision;
(3) the absence of restrictions to the number of alternatives and criteria that characterize the research object;
(4) the ability to prioritize alternatives by their proximity to the ideal solution;
(5) the consideration of the competence level of experts, who participate in decision-making;
(6) the consideration of the hierarchal structure of criteria that describe alternatives;
(7) the description of linguistic variables' values as fuzzy trapezoidal numbers, which ensure the robustness of criteria to the confidence interval boundaries;
(8) the possibility to extend the applied methodological approach to all HRM tasks that require intelligent support of decision-making.

The paper presents a step-by-step demonstration of the capabilities of the suggested method in multiple-criteria analysis and selection of decisions, by the example of the selection and recruitment, which is of methodological value. Alternative calculations for decision-making, based on the scoring and comparative analysis of the two methods' results, shows the efficiency of the suggested method.

The use of the described methodological approach as the mathematical basis for the computer system of decision-making support in HRM tasks can become an effective instrument for preparing and making efficient decisions in human resource management.

Human resources firmly connect the management system with the development strategy of any organization and play an important role in achieving its goals. The proposed method can be successfully used for HRM in various fields, particularly in: industry [20]; health care [39]; education [3, 4, 37]; public administration [42]; business sector, financial and banking sector [32], [33], [22]. In other words, the scope of application of the method covers all those spheres of human activity in which human resource planning, selection, evaluation and recruitment of personnel, career planning are important elements of their development. To solve these HR problems, it is necessary to apply scientifically confirmed approaches based on multicriteria decision-making methods.

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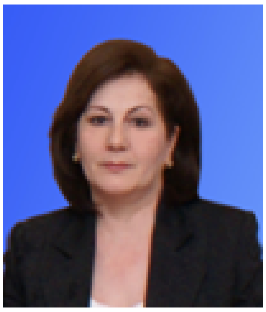

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